

## Topical Review

# A review of modelling and data analysis methods for accelerated test

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Received 22 December 2024, revised 11 February 2025

Accepted for publication 16 February 2025

Published 3 March 2025



CrossMark

## Abstract

High reliability and long service life have become the development goals and urgent needs of equipment research and development, especially for important equipment and major projects, which put forward new challenges to the traditional reliability technology. As an effective means to support the high reliability and long life of equipment, accelerated testing (AT) technology has become a hot research topic. On the basis of AT application demand analysis, the technical system of AT is proposed. The current state of research on modelling and analysis methods for AT is reviewed and analysed from five aspects: modelling and analysis of accelerated life tests (ALTs) with a single failure mode, modelling and analysis of accelerated degradation tests (ADTs) with a single failure mode, modelling and analysis of ALTs with multi-failure modes, and modelling and analysis of ADTs with multi-failure modes. And finally, future research directions for techniques related to modelling and analysis of ATs are envisioned.

Keywords: accelerated test, modelling analysis, accelerated life test, accelerated degradation test, reliability

## 1. Introduction

With the progress of science and technology, the reliability and life of the product requirements are getting higher and higher. At the same time, the contradiction between the user's expectation of high reliability and long life of the equipment and the

equipment manufacturer's requirement of short development cycle and low development cost is becoming more and more prominent. Therefore, the efficient growth and evaluation of equipment life and reliability has become an urgent problem to be solved. The emergence of accelerated test (AT) technology makes it possible to solve this problem.

The basic principle of AT technology is to accelerate the performance degradation or failure process of the specimen by increasing the test stress level under the premise that the failure mechanism remains unchanged. On one hand, the failure time or performance degradation data under accelerated conditions are obtained, which are converted to the normal (on-site) state by mathematical modelling to validate or evaluate or predict equipment life under normal stress level [1–4]. On the other

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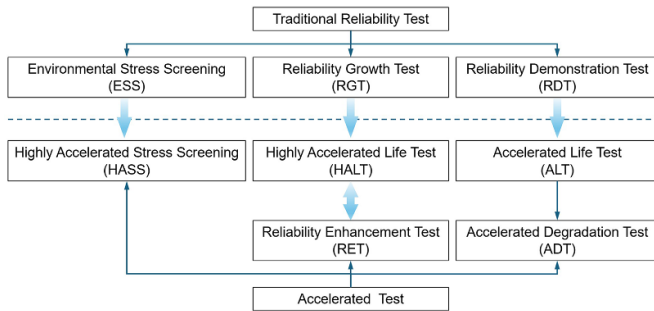


Figure 1. Accelerated test vs. traditional reliability test.

hand, product defects are quickly stimulated and thus to realize highly efficient reliability growth through design or process improvement on the one hand [5].

The relationship between AT and traditional reliability testing methods is shown in figure 1. Highly accelerated stress screening corresponds to traditional environmental stress screening (ESS), which is mainly applied in the production stage to quickly expose various manufacturing defects of products in the production process and eliminate products with early defects. Highly accelerated life test (ALT) is applied to the development stage to realize efficient reliability growth, which is called reliability enhancement test by Boeing when applying this technology [6, 7], and this term is widely used at present. Reliability enhancement test corresponds to the traditional reliability growth test, which can provide highly reliable growth technology for high reliability and long life engineering.

ALT corresponds to traditional reliability demonstration test. An ALT is a test method based on sound engineering and statistical assumptions which utilizes a statistical model related to physical laws of failure to transform reliability information generated in a short time by an economical and accurate method to quantitative repeatable estimates of a part's reliability characteristics when it is operated at rated stress levels [8]. Accelerated degradation test (ADT) is a developmental branch of ALT, which analyses degradation data of equipment performance. This test can overcome the difficulties of zero failure in the application of ALT and has a broad application prospect. ALT and ADT provide long-life prediction and validation techniques for high-reliability and long-life products. Due to the use of accelerated stress, the life course of the product is greatly compressed, and thus early assessment can be realized before the expiration of the product's service life, and the prediction ability is strong. Therefore, AT technology has become an inevitable requirement for product life assessment under time and cost constraints.

This paper is focused on ALT and ADT techniques, the purpose of which is the assessment and prediction of product reliability and life. AT appearing in the following text refer to ALT and ADT unless otherwise specified.

AT can be categorized into ALT, ADT, AT with a single failure mode, and AT with multi-failure modes according to the type of failure mode of the product under test. ALTs are

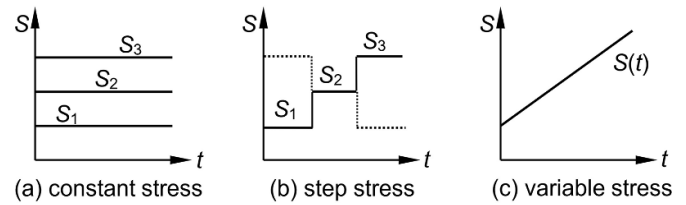


Figure 2. Types of stresses for accelerated tests.

mainly for products with sudden failure mode, in which the data obtained are failure time data. ADT is mainly for products with degradation failure mode, in which the data obtained are performance degradation data. In the AT with a single failure mode, only one failure mode occurs. In the AT with multi-failure modes, two or more failure modes occur in the product. The AT for this type of product is called AT with competitive failure modes, in which the test data may have both failure time and performance degradation data. ALT and ADT can be regarded as special cases of ATs with competitive failure modes.

According to loaded stress with time in the AT, the AT can be divided into constant stress AT (CSAT), step stress AT (SSAT), and variable stress AT (VSAT) [2], as shown in figure 2. SSATs include step-up stress ATs (shown as a solid line in the figure) and step-down stress ATs (shown as a dashed line in the figure).

The constant stress acceleration test is relatively simple to perform, analyse data, and the life assessment results are more accurate. SSAT can make the sample failure faster, and the required sample size is less, but the accuracy of the life assessment results is not as accurate as the constant stress test. VSAT is very demanding on equipment, and the data analysis is very complex. So it is rarely used in practical applications. SSAT can be considered as a special type of VSAT.

After the AT technique was proposed, it has received a great deal of attention due to its outstanding advantages in reliability and life assessment and validation, and the researchers' studies have mainly addressed the following issues:

(1) Modelling and data analysis of AT

Modelling and analysis methods of AT occupy an important position in AT research. Many researches in this field focus on this issue, mainly studying the data analysis methods of ATs for products with different failure types, such as sudden failure, degradation, single failure, multiple failure, etc. The main questions are studied including how to improve the accuracy of reliability and life assessment, and how to make the methods easy to apply in engineering practice.

(2) Validity of the AT

The validity of the AT refers to that test data of products can be obtained in a relatively short period of time by the AT to achieve product lifetime assessment, and there are good agreement between the results of the lifetime assessment and its actual operational lifetime. The issue of validity is currently a hot spot in AT research and application, as well as a bottleneck in solving the engineering applications of ATs in a wider

range of fields. In this problem, it is necessary to investigate how to check the consistency of the failure mechanism of the specimen in AT with that in service conditions to ensure that the modelling analysis prerequisites are valid.

### (3) Optimal design of AT

The optimal design of the AT is the inverse problem of the modelling analysis, which studies how to design the test under the given constraints (stress range, test cost, etc) in order to obtain the accurate estimation of various parameters of reliability. Optimal design can be described as a constrained extremum problem. The optimization objective is the accuracy of parameters estimation of reliability under use conditions. The constraint is the test cost including sample size and test time, etc

### (4) Engineering applications of AT

It has become a challenge to be solved how to use AT modelling analysis method to achieve its life prediction and reliability assessment in engineering practice due to complex structure and diverse service conditions of products. It is necessary to research and solve a series of problems such as how to design the AT program for a specific product object, how to carry out ATs to obtain test data, how to model and analyse the data to obtain the results of the prediction and evaluation of reliability and life, how to analyse the validity of the results, and so on.

This paper mainly reviews the current research status of modelling and analysis of ATs. Section 2 summarizes the research related to modelling and analysis of ATs with a single failure mode. Section 3 summarizes the current research status of modelling and analysis of ATs with multi-failure modes. In section 4, future research trends for techniques related to modelling and analysis of ATs are envisioned. Section 5 concludes the paper.

## 2. Modelling and data analysis of AT with a single failure mode

### 2.1. Modelling and data analysis of ALT with a single failure mode

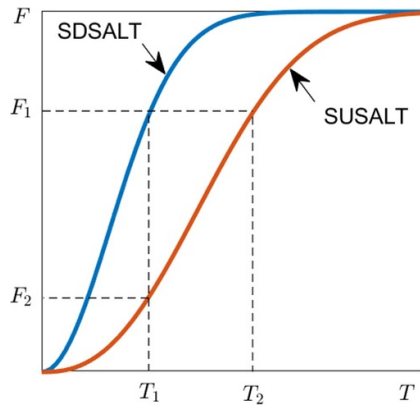
**2.1.1. Modelling and data analysis of CSALT with a single failure mode.** Due to the relative simplicity of the CSALT method, modelling analysis of CSALT with a single failure mode was first studied. It mainly focuses on how to improve the accuracy of statistical analysis. Hirose studied the nonlinear problem in the accelerated model, and introduced the threshold stress in the accelerated model to improve the model, which improved the accuracy of the constant stress test analysis [9]. Watkins investigated analysis method of the maximum likelihood estimation (MLE) for CSALT under Weibull distribution and constructed a simplified numerical solution model for easy engineering applications [10]. Bugaighis investigated the effect of the type of censorship (time censorship or type-I censorship, failure censorship or type-II censorship) on the MLE of the Weibull log-linear model [11]. McLinn introduced constraints between the life distributions

at each stress level, which led to an improvement in the accuracy of the statistical analysis [12]. Wang and Kececioglu further discussed the parameter constraint problem and proposed three models, 0, I and II, to model the parameter constraints and established the WK-MLE numerical solution method [13]. Kumar and Nassar established analysis methods for cases where the lifetime of a product at a constant stress level follows the generalized inverse Lindley distribution [14, 15]. Wu *et al* discussed interval estimation of the two-parameter exponential constant stress ALT model under Type-II censoring [16]. Smit and Lv proposed Bayesian analysis methods for the Eyring–Weibull model and constrained randomization in CSALT, respectively [17, 18]. Zhang *et al*, proposed a two-step analysis method based on constructive data for CSALT under Weibull distribution to solve the problem of the sensitivity of the initial value of the iterative algorithm for MLE, which improves the accuracy of modelling and analysis and avoids the process of table checking, and is easy to be applied in engineering [19].

**2.1.2. Modelling and data analysis of SSALT with a single failure mode.** The original step stress test method was the step load method applied in mechanical durability tests (e.g. fatigue tests) [20]. The advantage of the step-stress test is that this AT method reduces the requirement for the number of specimens and has a higher acceleration efficiency than the constant stress test.

With the exception of the initial stress level of the test, the single failure mode of a test unit from a step-stress test is generally the result of the combined effect of several different accelerated stress levels. So how to separate the product life information under each accelerated stress level from such failure data is a key issue in the modelling analysis of step-stress tests. In 1980, Nelson proposed the famous cumulative exposure model (CEM), also known as the Nelson CEM, Nelson's model or Nelson's principle [21]. According to the CEM, the test times of products under different accelerated stress levels can be converted to each other, thus making a breakthrough in the statistical analysis of step-stress test. At the same time, Nelson also statistically analysed the failure data of cable insulation materials under step-stress test, and obtained the estimation of life distribution parameters under normal stress level through the MLE method.

On the basis of Nelson's theory, the research work on the modelling analysis method of the SSALT has made great progress. Tyoskin and Krivolapov studied the interval estimation of parameters of the SSALT, and established a nonparametric model for the solution [22]. Tang and Sun proposed the linear CEM (LCEM) to improve the Nelson model to solve the problems of no-failure data in step-stress level, and investigated the application of LCEM in the analysis of SSALT [23]. Mao gave a method for statistical analysis of SSALT under exponential distribution by utilizing the properties of ordered statistics in exponentially distributed occasions [24]. Ma and Li, studied the statistical analysis of SSALT under lognormal distribution, and improved the MLE statistical analysis method of SSALT, which overcame the difficulty of MLE to find



**Figure 3.** Cumulative failure probability function for test samples in SDSALT and SUSALT respectively.

the multivariate function's optimum value, and made the analysis steps flowed [25]. Wu and Cheng studied the statistical analysis of SSALT with Weibull distribution, and discussed the deterministic and uncertainty relationship in the analytical problem, which improved the complexity of the algorithm [26]. Xu and Fei studied the modelling and analysis method of censoring type II SSALT with Weibull distribution and acceleration model of inverse power law, and obtained the point and interval estimation of the model parameters, and conducted a simulated comparative study on the analytical accuracy of the MLE and the inverse moment estimation [27]. Wang proposed a nonlinear mixed effect model with the assumption of Weibull distribution for life time data analysis from SSALT to consider the random effects from the group-to-group variation, in which both MLE and Bayesian inference are introduced for the estimation and prediction of model parameters as well as other statistics of interest [28]. Smith *et al* proposed a novel binomial damage model for ALT, which establishes the relationship between damage or life and stress through the binomial probability parameter. Specifically tailored to address failures caused by discrete (shock-based) stresses during a product's life, such as the dropping of a handheld device, this model provides a probabilistic framework to capture these specific occurrences [29].

On the basis of SSALT method, Zhang proposed the step-down-stress ALT method, and thoroughly studied the test efficiency, modelling and analysis methods and carried out experimental validation, which showed that the step-down-stress ALT can further improve the ALT efficiency significantly while ensuring the accuracy of the reliability estimation of the product [30]. Here the test efficiency is measured by the test duration  $t$  under the same test setting. The shorter  $t$  is, the higher the efficiency. The test efficiency of the step-down stress ALT (SDSALT) method is significantly better than that of the step-up stress ALT (SUSALT) method for the same test stress level setting with equal sample size, and equal censoring failures under each stress levels, as shown in figure 3. From the figure, it can be seen that the time ( $T_1$ ) required for the SDSALT to achieve the same failure probability is significantly less than the time ( $T_2$ ) for the SUSALT.

In the modelling and analysis of the SSALT, the complexity of the algorithm and the non-processing are the main problems at present. These two problems make the engineering application of the SSALT more difficult and make it difficult to realize the software programming of the statistical analysis, which reduces the value of its engineering application.

**2.1.3. Modelling and data analysis of VSALT with a single failure mode.** The loading stress in VSALT changes continuously with time, which can stimulate the specimen to fail faster, thus further improving the efficiency of the ALT. The simplest VSALT is the progressive stress ALT (PSALT). Due to the advantages in acceleration efficiency, the PSALT method has been gradually applied to the ALT, and has developed into a basic test method.

Yin and Shen derived the life distribution function of product under PSALT when the life distribution under design constant stress follows an exponential or a Weibull distribution and gave the MLE of the parameters [31]. It is pointed out in [31] that the MLE of the PSALT does not necessarily exist, and the test data are required to satisfy certain constraints even for exponentially distributed occasions. Lin and Fei gave the statistical inference of the PSALT for the Weibull and lognormal distributions, which applied on solid tantalum electrolytic capacitors [32]. Ma and Ge deduced the life distribution function of product under PSALT under lognormal distribution, and gave the statistical analysis method for this occasion [33]. Wang and Wang also studied the statistical analysis of PSALT with lognormal distribution [34]. Xu discussed the parameter estimation of multi-group of PSALT with Weibull distribution using a rather simple calculation method [35]. In response to the problem of unsolved cases in the analytical method in [32], Tang and Fei proposed a new method for the parameter estimation of the PSALT and developed a corresponding software package in C language [36].

Products in most equipment generally are exposed to cyclic stresses during operation or storage stage. For instance, car bearings experience repetitive forces while in use; aircraft electronics are exposed to fluctuating temperatures because of changing sunlit and shaded regions; and outdoor mechatronic devices face temperature fluctuations due to daily and seasonal cycles. Clearly, it is illogical to evaluate the lifespan and reliability of these products through the application of the aforementioned fundamental ALT techniques.

A cyclic stress ALT (CySALT) offers a more precise and efficient approach for predicting the lifespan and reliability of products under cyclic stress conditions. In this test, the specimen is subjected to recurring stress cycles. CySALT has dual benefits: (1) it can mimic the real-world cyclic stress patterns that products endure, using these cycles as an acceleration factor. This enhances the validity of the test results and improves the accuracy of the predictions. (2) By fine-tuning the cyclic stress parameters, CySALT can be conducted more efficiently, which can substantially decrease both the cost and duration of the testing process.

Cui proposed a temperature CySALT method coupled with the Coffin–Manson-based model to offer a robust framework for predicting the reliability of electronic equipment packaging [37]. By quantifying the acceleration factor and activation energy for via cracking, this approach enables more informed decisions in the design and testing of electronic packaging to mitigate failure risks.

Shohji *et al* performed thermal cycling tests on chip-level package solder joints under various conditions, revealing that microstructural coarsening and solder joint fractures were observed at temperatures reaching approximately 100 °C, with the thermal fatigue life adhering to the modified Coffin–Manson equation [38]. In contrast, when the maximum temperature was room temperature and the temperature range was minimal, solder joint fractures occurred without microstructural coarsening, and the thermal fatigue life deviated from the modified Coffin–Manson equation. Zhu *et al* proposed a method of reliability estimation for one-shot devices under CyALT by the integration of the Norris–Landzberg model with the Birnbaum–Saunders distribution offering a more nuanced understanding of their performance under varying cyclic temperature conditions [39].

Wang studied CyALT method for mechatronic products including test protocol and its critical parameters, methodology for modelling and analysing CyALT data, and a case study, which can offer technical guidance for assessing the lifespan and reliability of mechatronic products under conditions of cyclic stress [40].

Compared with the CSALT and the SSALT, the acceleration efficiency of the VSALT is undoubtedly the highest, but its statistical analysis is the most complicated, and its application is greatly limited. In addition, the VSALT requires a specialized device to generate the required stress, which makes the application difficult.

## 2.2. Modelling and data analysis of ADT with a single failure mode

In ADTs, the amount of product degradation at different moments can be obtained, and when the amount of degradation exceeds a specified threshold, the product is considered to have failed. Therefore, these degradation data can provide useful information for estimating product reliability. The use of degradation data instead of failure time data for reliability or life assessment has the following advantages: (1) for many products, degradation is a natural attribute, and degradation data can be obtained by monitoring their performance data regardless of whether or not a failure occurs; (2) degradation data can be applied to situations where there are only a few failures or zero failures, and can provide more information than the failure time data (reduced information lost from failure time data); (3) ADT can yield more accurate life estimates than ALT with very few failures or zero failures; and (4) degradation data can provide more information about the degradation process, which can help us to discover a mechanistic model between degradation and accelerated stress [41].

Accelerated degradation modelling assumes the following: (1) the progress of product's degradation is irreversible; (2)

one accelerated degradation model corresponds to one degradation process, mechanism, or failure mode; (3) the performance of test specimen prior to the start of ADT can be ignored; (4) the failure mechanism under high stress level is the same as that under the design or conventional use stress level [2, 42].

According to the degree of understanding of the degradation mechanism of product performance, accelerated degradation models can be categorized into two basic types: physics-based and statistics-based accelerated degradation models [43–45].

(1) Physics-based accelerated degradation models. If the degradation mechanism of a product is well understood, then a physics-based accelerated degradation model can be developed to evaluate the reliability of the product. This kind of model contains some parameters which are random variables, and the derivation of the reliability function through these random variables may be very difficult or complicated. So the analytical form of the reliability function expression can not be obtained in general for this kind model.

Nelson analysed the degradation data of an insulating material under different stress levels, and used the accelerated degradation model to describe the relationship between the absolute temperature of the material and the breakdown voltage versus time [46]. Carey and Koenig assumed that the relationship between the maximum degradation and the absolute temperature of an integrated logic family can be described by an Arrhenius model, and then estimated the model parameters using a MLE approach, and finally used this model to predict the maximum degradation of a product operating at room temperature enabling reliability assessment of the integrated logic family [47].

Tseng analysed the manufacturing process of a mercury lamp and determined that the main factors affecting its degradation were the concentration of mercury in the lamp and the concentration of argon gas, and obtained an accelerated degradation model for this mercury lamp [44]. Meeker and Hamada discussed the general model of univariate degradation process, and they classified univariate degradation processes into three types: linear degradation, convex degradation, concave degradation [48]. Feinberg and Widom studied the Arrhenius theorem in depth and gave a time-dependent model of thermally activated energy to relate the model parameters to the catastrophic failure, which showed that the Arrhenius degradation behaviour generally obeyed the lognormal distribution [49]. Chuang *et al* described the kinetic modelling of LED degradation [50]. Meeker *et al* discussed the degradation model and method based on degradation data to derive product reliability and compared the degradation test method with the traditional life test method [51].

(2) Accelerated degradation models based on statistical data. The statistical data-based accelerated degradation model is more applicable in engineering because it describes the accelerated degradation data with a statistical model. However, since the statistical data-based model is generally based on the assumption that the degradation trajectory curve is linear and the standard deviation of the degradation distribution is constant, the statistical data-based degradation model is only valid for a constant stress level. Lu and Meeker

employed a nonlinear mixed-effects model and applied a two-step analytical approach to calculate the time-to-failure at various percentiles [52]. Additionally, they leveraged Monte Carlo simulations to determine point estimates and construct confidence intervals for the reliability assessment. They gave a general trajectory model that includes fixed-effect parameters that are applicable to all products and random-effect parameters that only describes the characteristics of degradation of individual products. Yang and Yang proposed a method of ADT with tightened critical values by considering more life data can be obtained by tightening the critical value [53]. They modelled the relationship between life and critical value and stress to estimate the life distribution at a usual critical value and design stress by MLE. This approach assumes that the degradation trajectory is linear and the degradation follows a normal distribution. Lu gave a model with random regression coefficients and standard deviation functions to analyse linear degradation data from semiconductors [43]. Tang and Chang developed a model for non-destructive accelerated degradation data which were collected from power supply units [54]. Tseng *et al* used a simple linear regression with random coefficients to model the luminosity degradation of fluorescent lamps [55].

Many scholars have paid much attention to the modelling and analysis of degradation data. Nelson studied the life time distribution based on the performance degradation relationship, and gave a complete description of the ADT, including the scope of application, statistical modelling, and data analysis methods [2]. Nachlas introduced a comprehensive model for predicting degradation acceleration under thermal cycling conditions, potentially aiding in the development of ADT and ESS methodologies [56]. However, empirical data and practical case studies are required to affirm the model's accuracy.

Meeker *et al* gave the analysis methods of accelerated degradation data, and they assumed that the degradation data can be described by a mixed-effects nonlinear regression model, and used approximate MLE to estimate the model parameters, and proposed a method for estimating the lifetime time distribution in different cases [42]. A nonparametric regression accelerated lifetime stress modelling analysis method was given by Shiau and Lin [57]. The method assumes that the changing stress level only affects the performance degradation rate and not the shape of the degradation curve, thus weakening the assumption of the regression function form and allowing the accelerated degradation data to play a dominant role in the fitting process of the accelerated degradation model.

Aiming at the problem that the theoretical derivation of the MLE method for ADT with single failure mode is complicated which is difficult to be applied to the engineering, Deng proposes the constant stress ADT methods based on the degradation amount distribution and based on the pseudo-failure life, respectively, and the study shows that the proposed methods have high accuracy in the estimation of the reliability and the life, and are easy to be applied to the engineering application [58–60].

In summary, the main limitations of current modelling and analysis methods for single failure mode AT lie in the assumption that products fail solely due to a single mechanism. In reality, products may exhibit multiple concurrent

failure modes (such as corrosion, fatigue, wear, etc), leading to deviations between model predictions and actual failure behaviours. These methods also neglect the competition and synergistic effects between failure modes, as multiple potential failure mechanisms might accelerate or delay the primary failure process, yet the models fail to account for these interactions. Key challenges include: ensuring that model structures (e.g. acceleration equations, degradation paths) accurately reflect physical failure mechanisms rather than serving as mere mathematical fits. Existing models, primarily designed for constant stress conditions, struggle to handle dynamic stresses (e.g. temperature cycling, random vibrations) encountered in real-world applications. Many models assume linear or stationary failure processes, whereas actual failures may exhibit threshold characteristics (e.g. material yield points) or abrupt transitions (e.g. brittle fractures). Future advancements could focus on: integrating physical mechanisms with data-driven approaches to enhance model interpretability, developing dynamic accelerated models capable of simulating time-varying stress profiles, incorporating advanced uncertainty quantification techniques to better characterize model reliability.

### 3. Modelling and data analysis of AT with multi-failure modes

#### 3.1. Modelling and data analysis of ALTs with multi-failure modes

Products in service often have multiple failure modes, their failure is generally caused by these multiple failure modes compete with each other, that is, the result of competitive failure. Competitive failure is an important form of product failure, and the statistical analysis method of ALT for products with competitive failure is very different from that of single failure occasions. And it is of great significance to study the statistical analysis of ALT for competitive failure occasions.

According to the existence of correlation between the competitive failure modes, competitive failure can be divided into independent and competitive failure occasions. In ALT with independent competitive failure, it assumes that the competitive failure modes of the product are independent of each other, there is no correlation. However, in ALT with dependent competitive failure, it assumes that the competitive failure modes of the product are not completely independent of each other, and there is a certain degree of correlation between each other.

The traditional statistical analysis of AT usually assumes that the product has only one failure mode. With the continuous development of AT technology, its application areas have been expanded from component-level or material-level products with single function, single structure and a single failure mode to assemblies-level products with a high degree of integration and a variety of failure modes. The products in AT have often more than one failure mode which affects with each other. When each of the many failure modes of a product can lead to failure, that is, the lifetime of the product is the smallest failure time in each failure mode, the product is said to be a competitive failure product or product with competitive

failure modes [61]. For example, in the ALT of insulation system of an electric motor, the failure of the insulation system is generated by one of failure modes: turn-to-turn failure, phase-to-phase failure, and slot-to-slot failure, and the life of the insulation system is the minimum failure time of these three failure modes. Bearing failure can result from one of failure modes: inner ring failure, outer ring failure, rolling element failure, and cage failure. Semiconductor device failures can be attributed to electrical failures or mechanical fatigue of I/O interfaces (e.g. solder joints, etc) exhibiting competing failure modes. A breakdown short-circuit in the motor coil can cause a sudden motor failure, while wear and tear of the mechanical components can cause a degradation failure, and in practice the form of motor failure is the result of the competition between these two failure modes. Thus, it is of great significance to analyse the product reliability based on the competing failure modes, and to consider multiple competing failure modes to model and analyse the reliability of the product more in line with the actual failure law of the product [62].

The modelling and analysis methods for ATs of products with competitive failures are very different from those for single failures, and it is of great significance to study the modelling and analysis of ATs for competitive failure scenarios. Nelson investigated the series system model of competitive failures, and proposed the graph analysis method and the maximum likelihood analysis method for the statistical analysis of ATs [2]. Klein and Basu studied the statistical analysis methods for ALTs with competitive failures in the case of exponential distribution and Weibull distribution [63]. Kim and Bai investigated the estimation of life distribution for constant stress ALT under stress conditions for two competitive failure scenarios, namely intrinsic failure and exogenous failure, and gave the estimation process based on EM algorithm, and the specific formulae were given for the Weibull distribution [64]. Pascual investigated the optimal design of ALT with competitive failure when the failure modes obey the Weibull distribution [65]. Bunea and Mazzuchi proposed a Bayesian model for the statistical analysis of ALT with competitive failure to solve the problem that the MLE method is difficult to be applied due to little failure data in the test [66, 67].

Zhang and Mao investigated the statistical analysis of simple SSALT for products with competing failure mechanisms under exponentially distributed occasions, and studied the effect of failure mechanisms on the estimation of mean time to failure under use stress [68]. Wang and Wang studied the analysis model and point estimation of its parameters for PSALT with competing failure mechanisms under exponential distribution, and derived the inverse moment estimator and the maximum likelihood estimator for the parameters [69]. Zhang investigated the data analysis method of ALT for products with competitive failure mechanisms under exponential and Weibull distributions, discussed the influence of failure mechanisms on the estimation of reliability parameters under use stress level, and compared and analysed the results between the cases of differentiated failure modes and non-differentiated failure modes [70]. Wang and Wang discussed the parameter estimation of the CSALT and SSALT for products with competing failure mechanisms under exponential distribution, and

improved the commonly used MLE method, and illustrated the effectiveness of the improvement with the results of the large samples and the simulation analysis [69, 71]. Gao used mixed distributions to model and analyse the competitive failure problem, but the mixed distributions can only be used in the case where the failure times of the failure modes have the same family of distributions (e.g. exponential distribution) [72].

The current researches about analysis method of AT with competing failure are mostly based on the assumption that competing failure modes are independent of each other. However, failure modes of products are generally dependent of each other due to common working and environmental conditions and other coupling factors. For example, bearing failure can be considered as the result of failure competing between the outer ring failure, inner ring failure, cage failure and rolling body failure. These four units are coupled together in the design, manufacturing, assembly and work process by the structure of the size of the mutual matching, interrelated movement parameters, etc. The unit life model or function contains the same random variables, so the failure of the unit is dependent with each other. The assumption that each failure mode of the product is independent of each other is inconsistent with the actual situation, and there will be a large gap between the statistical analysis results and the actual situation, making it difficult to accurately estimate the life and reliability of the product. Therefore, it is of great significance to study the statistical analysis method of AT with competing failure to improve the accuracy and efficiency of AT.

One of the key issues is how to relate the failure time distribution of products with competing failure to the marginal distribution function of the potential failure time of each failure mode. Some possible methods include the joint distribution method, the conditional correlation coefficient method, the linear regression method, and the Copula method [73]. In comparison, Copula method is more convenient and has become an important tool for solving the correlation problem, and has gradually become a current research hotspot, penetrating into the correlation research in various fields such as risk management, financial market analysis, biomedical research and other fields [74–80].

To address the problem of modelling and analysing the ALT of bearings with multiple failure modes, Zhang firstly studied the statistical analysis method of the CSALT, VSALT, SSALT with independently competing failure and established the maximum likelihood model for parameter estimation. At the same time, Zhang proposed a parameter estimation method based on the artificial fish swarm algorithm for the characteristics of multiple parameters and complex models in the statistical analysis of ALTs with competitive failure. The results of the application examples show that the method is easy and simple to apply, with good estimation accuracy, which provides a highly efficient tool for solving the problem of multi-parameter estimation in the statistical analysis of ALTs for competitive failure occasions [81, 82].

Subsequently, he proposed a new multi-dimensional Archimedean Copula construction method for the analysis of AT data in competitive failure-related occasions by synthesizing the features of the existing construction methods

[83]. This method has a simple process, and avoids the complex conditional probability density calculation and multiple integrals. At the same time, it can conveniently represent the correlation between different combinations of variables at a lower level. And then he proposed the statistical analysis models of CSALT and SSALT based on Copula model.

Li *et al* investigated and proposed a new AT method to estimate the functional lifespan of complex electronic devices with multiple failure mechanisms by simulating thermal cycles and impact stress [84]. This method improves upon traditional approaches by better reflecting failure mechanisms sensitive to low temperatures and has been validated through real-world testing on custom circuits.

Pan proposed a multi-stress coupling acceleration model for reliability evaluation of complex electromechanical products with high reliability and long lifespans [85]. It incorporates a physics of failure model based on a Copula function and a coupling matrix to analyse multi-stress interactions, offering improved accuracy and broader applicability compared to traditional methods.

### 3.2. Modelling and data analysis of ADTs with multi-failure modes

Zhao and Elsayed analysed the ADT in the presence of sudden-type failure, and described the degradation path by Brownian motion, and statistically inferred the reliability under use stress by the method of MLE [86].

Aiming at the problem that the AT with competing failure mainly focuses on the constant stress but lacks related research on the step stress occasions, Tan *et al* modelled and analysed the SSAT for competitive failure occasions in the most general form (the co-existence of sudden and degradation type failures) [87]. The non-completeness of the test data due to the uncertainty of the failure modes corresponding to failure samples and the test censoring is fully taken into account. And he proposed a method to statistically analyse failure-time data with sudden failure mode and degradation failure mode by MLE based on expectation-maximization algorithm and MLE of the pseudo-failure-life, respectively.

Zhang proposed a method for statistical analysis of ADT for competitive failure-related occasions based on copula model [88]. He established methods of statistical analysis of ADT with competing failure based on approach of degradation distribution and pseudo-failure-life, respectively.

Cao *et al* developed a proportional hazard-based hard failure model by incorporating hard failure and correlation of the degradation process with stress levels to address the challenge of assessing competing failure reliability in ADT [89]. The parameters in the model were estimated using MLE and linear regression. This approach was validated through an example, demonstrating its effectiveness in modelling the constant stress accelerated degradation problem.

Liu *et al* proposed a reliability modelling and a statistical inference method of ADT with multiple stresses and dependent competing failure processes [90]. It incorporates the coupling of multiple stresses in an acceleration model and models the degradation-shock dependence using the Facilitation

model. The model's accuracy is validated through a practical example, showing that it can accurately predict lower reliability under higher environmental stresses, and that the parameter estimation error is minimal at 0.34%.

Chi *et al* proposed a reliability assessment approach for the micro inertial measurement unit (MIMU) with long-lifetime and multiple degradation, based on ADT data and Copula theory [91]. Each degradation model of MIMU is constructed by drift Brownian motion and the Copula function is used to model the dependency between multiple MIMU performance parameters. The particle swarm optimization algorithm is used to estimate the unknown parameters in the multi-dependent ADT model.

In summary, the current limitations and challenges in modelling and analysis methods for AT with multiple failure modes lie in the constraints of model assumptions. Existing approaches typically assume independence between different failure modes, whereas actual systems may exhibit correlations among failure modes. Multi-failure-mode systems might involve multiple performance levels or intermediate states (e.g. partial failure, performance degradation), which cannot be adequately described by traditional binary-state models (normal/failure). Different failure modes in the same unit might trigger distinct types of system failures (e.g. Type I requiring all units to fail vs Type II requiring only single unit failure), necessitating modelling through Markov state transition diagrams. However, the state space grows exponentially with the number of units, making analytical solutions challenging. Future research could explore hybrid models considering both failure mode correlations and temporal dependencies, such as combining Copula functions with Markov chains. Advancements in multi-failure-mode AT modelling require breakthroughs in model assumptions, computational efficiency, and data validation, while integrating emerging technologies to drive methodological innovation, thereby meeting the engineering requirements for reliability analysis of complex systems.

## 4. Development trend of modelling and data analysis of AT

At present, AT theory and method has made great progress, and has been successfully applied in many engineering practice. However, the AT technology is still in the stage of vigorous development, and there are many theoretical innovation and application of the transformation of the problem needs to be solved. The development trend of AT technology is mainly as follows:

- (1) Multi-SSAT modelling and analysis. Products in the actual service process are usually subject to a variety of stresses at the same time. So a single stress in AT can not truly reflect the actual service state of the product. The use of multiple stresses as the AT stresses of the equipment can fully reflect the service conditions to obtain more accurate life prediction with a larger acceleration factor and higher test efficiency. Modelling and analysis, optimization design,

and engineering application of multi-SSATs are expected to become hot research topics. Multi-SSAT involves a combination of multiple stresses (e.g. temperature, humidity, voltage, etc), and there may be complex interactions between these stresses, increasing the difficulty of modelling. More advanced statistical models and machine learning algorithms can be developed in the future to capture and quantify the interactions between multiple stresses. For example, generalized linear models or deep learning models are used to handle complex nonlinear relationships. In addition, designing an experimental program capable of covering multiple stress combinations while efficiently acquiring data is also a complex issue. It may be necessary to use experimental design methods, such as response surface methodology or Taguchi methodology, to optimize the test protocol to obtain the maximum amount of information with the minimum number of trials.

- (2) Modelling and analysis of AT for complex system. Research about AT is currently focused on occasion of a single failure mechanism as well as multiple failure mechanisms which is independent of each other. Complex systems often have multiple failure mechanisms that may be coupled with each other. The system failure is the result of these failure mechanisms competing with each other. The problem of AT with competing failure has attracted great attention from related scholars. Some research progress has been made in statistical analysis and optimal design of AT with multiple competing failure mechanism. The discussion of related issues will remain the focus of research attention in the coming period. Complex systems are usually composed of multiple interrelated subsystems, each of which may involve multi-physical field (e.g. thermal, force, electrical) coupling, dynamic interaction and nonlinear behaviour, and may also involve many different failure modes (e.g. corrosion, fatigue, material degradation), and traditional AT models are difficult to accurately describe the overall behaviour. In addition, the high cost and long cycle time of AT of complex systems lead to limited amount of test data, sensor noise, missing data, or difficulty in integrating heterogeneous data (e.g. structured and unstructured data). In the future, multi-scale and multidisciplinary modelling methods can be developed, using multi-scale modelling techniques, integrating layer by layer from microscopic (material level) to macroscopic (system level), and combining physical models (e.g. finite element analysis) and data-driven models (e.g. deep learning) to improve model accuracy. Synthesize missing data using generative adversarial networks to solve the data scarcity problem. Integrate multi-source data (experimental data, simulation data, historical data) to construct hybrid digital twins.
- (3) Accuracy of modelling and analysis of AT. AT is a developmental branch of statistical test, so the analysis accuracy is crucial for AT technology, which is also a common problem in statistical test research. The problem of statistical accuracy will remain a theme in the research of analysis methods of AT. In order to improve the accuracy of

AT modelling and analysis, utilizing more complex models, as well as combining physical models with data-driven models is a better idea. However, modelling using complex models (e.g. deep neural networks) may overfit limited data and reduce generalization capabilities. Difficulty in integrating physical models with data-driven models leads to a trade-off between interpretability and accuracy. In the future, physics-guided machine learning methods can be developed to embed physical equation constraints (e.g. conservation of energy) in neural networks to enhance extrapolation capabilities, while Bayesian inference can be considered to reduce estimation bias under small samples by utilizing prior knowledge (e.g. historical data) and quantifying uncertainty through Markov chain Monte Carlo sampling.

- (4) Engineering software for AT. With the AT technology research depth and maturity, research and development of Engineering software for AT will become an inevitable trend in this field of research. Currently, analysis software such as ALTA, Weibull++, Minitab, ReliaSoft has emerged, which possess basic data analysis capabilities. However, some of the latest research findings are yet to be transformed into engineering software. AT involves multiple disciplines such as material science, statistics, computer science, etc, and software development requires the integration of cross-disciplinary expertise. The computational complexity of AT models (e.g. multi-stress coupling models, nonlinear degradation models) is high, and traditional algorithms may be difficult to meet real-time requirements. Engineering software is usually oriented to professional users, with complex interfaces and steep learning curves, limiting the use of non-professional users. Models in the software require effective validation mechanisms, and users need to assess the accuracy and uncertainty of the models. In the future, modular design can be adopted to encapsulate different functions (e.g. data import, model construction, and result visualization) into independent modules, which is easy to extend and maintain. Support plug-in mechanism, allowing users to customize algorithms or integrate third-party tools. Utilize GPU acceleration or distributed computing to improve the computational efficiency of complex models. Use parallel algorithms to process large-scale data. Adopt drag-and-drop interface or natural language interaction to reduce user learning cost. Provide detailed tutorials and online help documents to support users to get started quickly. Integrates cross-validation, Monte Carlo simulation and sensitivity analysis to help users evaluate model performance. Provide uncertainty quantification results such as confidence intervals and prediction intervals.

## 5. Discussion

The optimal design of ATs is another research direction apart from the modelling and analysis of AT data. It mainly investigates how to design test schemes to obtain the most accurate

predictions of product life and reliability at the lowest experimental cost, which is essentially the inverse problem of data modelling and analysis. The objective function for the optimal design of ATs plans is generally a measure of the accuracy of life and reliability estimation, such as the estimation variance of the  $p$ -quantile life, mean square error, and the expected width of the confidence interval. The design variables of the plan are related to the type of AT, such as accelerated stress levels, sample size, test monitoring intervals and frequency, and test duration. The constraints are usually the total cost or total time of the test. Numerous scholars have conducted research on the optimal design of AT, covering the optimization of test schemes for constant-stress [92–94], step-stress [95, 96], and varying-stress [97, 98] ALTs as well as ADTs. This remains a hot research topic in the field [99–104].

## 6. Conclusion

Modelling and analysis technology of AT is the current research hotspot in the field of reliability mathematics and reliability engineering, and it has important application value for the life assessment of long-life and high-reliability products. This paper proposes a technical system of AT as shown in figures 1 and 2, and reviews and analyses the current research status of modelling and analysis methods of AT from five aspects: modelling and analysis of ALTs with a single failure mode, modelling and analysis of ADTs with a single failure mode, modelling and analysis of ALTs with multi-failure modes, modelling and analysis of ADTs with multi-failure modes. Finally, the future research direction of modelling and analysis technology of AT is prospected. With the further deepening of the research, the modelling and analysis technology of AT will have a wider application prospect.

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## References

- [1] Wang Y, Chen X, Zhang C and Zhang S 2023 *Modeling and Analysis Method and Application of Accelerated Testing of Electromechanical Products* (National Defense Industry Press)
- [2] Nelson W 1990 *Accelerated Testing: Statistical Models, Test Plans and Data Analysis* (Wiley)
- [3] Chen X, Zhang C, Wang Y and Tan Y 2013 *Accelerated Life Test Technology and Application* (National Defense Industry Press)
- [4] Tang L C and Cai J 2025 Accelerated reliability testing: from practice to theory *Reliability Analysis and Maintenance Optimization of Complex Systems (Springer Series in Reliability Engineering)* ed Q Q Zhao, I H Chung, J Zheng and J Kim pp 289–314 (Springer)
- [5] Wen X, Chen X, Zhang C and Tao J 2007 *Theory and Application of Reliability Enhanced Testing* (China Science Publishing & Media Ltd)
- [6] Chen X and Zhang C 2009 Research, application and development of accelerated testing *J. Mech. Eng.* **45** 130–6
- [7] Deppe R W and Minor E O Reliability enhancement testing (RET) *Proc. Annual Reliability and Maintainability Symp. (RAMS) (11 February 2025)* (IEEE) pp 91–98 (available at: <http://ieeexplore.ieee.org/document/291088/>)
- [8] Yurkowsky W, Schafer R E and Finkelstein J M 1967 Accelerated testing technology *Report No.: RADC-TR-67-420* (Rome Air Development Center)
- [9] Hirose H 1993 Estimation of threshold stress in accelerated life-testing *IEEE Trans. Reliab.* **42** 650–7
- [10] Watkins A J 1994 Review: likelihood method for fitting Weibull log-linear models to accelerated life-test data *IEEE Trans. Reliab.* **43** 361–5
- [11] Bugaighis M M 1995 Exchange of censorship types and its impact on the estimation of parameters of a Weibull regression model *IEEE Trans. Reliab.* **44** 496–9
- [12] McLinn J A New analysis methods of multilevel accelerated life tests 1999 *Annual Reliability and Maintainability Symp. 1999 Proc. (Cat No99CH36283) (27 September 2024)* (IEEE) pp 38–42 (available at: <http://ieeexplore.ieee.org/document/744094/>)
- [13] Wang W and Kececioglu D B 2000 Fitting the Weibull log-linear model to accelerated life-test data *IEEE Trans. Reliab.* **49** 217–23
- [14] Kumar D, Nassar M, Dey S and Alam F M A 2022 On estimation procedures of constant stress accelerated life test for generalized inverse Lindley distribution *Qual. Reliab. Eng. Int.* **38** 211–28
- [15] Nassar M, Dey S, Wang L and Elshahhat A 2024 Estimation of Lindley constant-stress model via product of spacing with Type-II censored accelerated life data *Commun. Stat.—Simul. Comput.* **53** 288–314
- [16] Wu W, Wang B X, Chen J, Miao J and Guan Q 2023 Interval estimation of the two-parameter exponential constant stress accelerated life test model under Type-II censoring *Qual. Technol. Quant. Manage.* **20** 751–62
- [17] Smit N, Raubenheimer L, Mazzuchi T and Soyer R 2024 A Bayesian generalized Eyring-Weibull accelerated life testing model *Qual. Reliab. Eng. Int.* **40** 1110–25
- [18] Lv S, Li F, Wang G and Li S 2024 Bayesian analysis of accelerated life test under constrained randomization *Qual. Eng.* **36** 105–17
- [19] Zhang C, Chen X and Li Y 2002 Analysis for constant-stress accelerated life testing data under Weibull life distribution *J. Natl Univ. Def. Technol.* **24** 81–84
- [20] Shiomi H 1982 *Fundamentals of Failure Physics* (China Science Publishing & Media Ltd)
- [21] Nelson W 1980 Accelerated life testing—step-stress models and data analyses *IEEE Trans. Reliab.* **29** 103–8
- [22] Tyoskin O I and Krivolapov S Y 1996 Nonparametric model for step-stress accelerated life testing *IEEE Trans. Reliab.* **45** 346–50
- [23] Tang L C, Sun Y S, Goh T N and Ong H L 1996 Analysis of step-stress accelerated-life-test data: a new approach *IEEE Trans. Reliab.* **45** 69–74
- [24] Mao S S 1985 Statistical analysis of accelerated life testing-step-stress models under the exponential distribution case *Acta Math. Appl. Sin.* **8** 311–6
- [25] Ma H and Li C 1996 Model and data analysis methods for step-stress accelerated life tests under lognormal distribution *Math. Appl.* **9** 39–41
- [26] Wu S and Cheng X 1999 Statistical analysis of stepwisely stress accelerated life test for the occasion of Weibull distribution *J. Huaqiao Univ. Nat. Sci.* **20** 109–13
- [27] Xu X and Fei H 1999 Statistical analysis of accelerated life testing-step-stress models under the Weibull distribution case *OR Trans.* **3** 73–84
- [28] Wang J 2020 Data analysis of step-stress accelerated life test with random group effects under Weibull distribution *Math. Probl. Eng.* **2020** 1–11

- [29] Smith R, Elsibaie S M and Modarres M 2024 Introduction to a binomial approach to accelerated life testing 2024 *Annual Reliability and Maintainability Symp. (RAMS)* (10 February 2025) (IEEE) pp 1–6 (available at: <https://ieeexplore.ieee.org/document/10457678/>)
- [30] Zhang C 2002 Theory and method of step-down-stress accelerated life testing *Degree of Philosophy Doctor in Engineering* National University of Defense Technology, Changsha, China
- [31] Yin X-K and Sheng B-Z 1987 Some aspects of accelerated life testing by progressive stress *IEEE Trans. Reliab.* **36** 150–5
- [32] Lin Z and Fei H 1991 A nonparametric approach to progressive stress accelerated life testing *IEEE Trans. Reliab.* **40** 173–6
- [33] Ma H and Ge G 1991 Statistical analysis of progressive stress accelerated life test under log-normal distribution *J. Hebei Normal Univ. Nat. Sci.* **2** 61–66
- [34] Wang B and Wang L 1995 Statistical analysis for the progressive stress accelerated life tests *J. East China Norm. Univ. Nat. Sci.* **4** 1–8
- [35] Xu X 1997 The method of parameter estimation in the progressive stress accelerated life testing *J. Shanghai Normal Univ. Nat. Sci.* **26** 36–41
- [36] Tang Y and Fei H 1998 Statistical analysis with software of progressive stress accelerated life testing under Weibull distribution *Appl. Math. J. Chinese Univ.* **13** 407–13
- [37] Cui H 2005 Accelerated temperature cycle test and Coffin-Manson model for electronic packaging *Annual Reliability and Maintainability Symp., 2005 Proc.* pp 556–60
- [38] Shohji I, Mori H and Orii Y 2004 Solder joint reliability evaluation of chip scale package using a modified Coffin-Manson equation *Microelectron. Reliab.* **44** 269–74
- [39] Zhu X, Liu K, He M and Balakrishnan N 2021 Reliability estimation for one-shot devices under cyclic accelerated life-testing *Reliab. Eng. Syst. Saf.* **212** 107595
- [40] Wang Y, Hu J, Zhang S and Chen X 2024 Cyclic stress accelerated life test method for mechatronic products *Qual. Reliab. Eng. Int.* **40** 1672–84
- [41] Stock D, Samanta P and Vesely W 1994 Development and application of degradation modeling to define maintenance practices. Nuclear regulatory commission *Report No.: NUREG/CR-5967* (Division of Engineering; Brookhaven National Lab) (<https://doi.org/10.1111/j.1432-1033.1994.tb19085.x>)
- [42] Meeker W Q, Escobar L A and Lu C J 1998 Accelerated degradation tests: modeling and analysis *Technometrics* **40** 89–99
- [43] Lu J-C, Park J and Yang Q 1997 Statistical inference of a time-to-failure distribution derived from linear degradation data *Technometrics* **39** 391–400
- [44] Yu H and Tseng S 2002 Designing a screening experiment for highly reliable products *Nav. Res. Logist.* **49** 514–26
- [45] Yang K and Yang G 1998 Degradation reliability assessment using severe critical values *Int. J. Reliab. Qual. Saf. Eng.* **05** 85–95
- [46] Nelson W 1981 Analysis of performance-degradation data from accelerated tests *IEEE Trans. Reliab.* **30** 149–55
- [47] Carey M B and Koenig R H 1991 Reliability assessment based on accelerated degradation: a case study *IEEE Trans. Reliab.* **40** 499–506
- [48] Meeker W Q and Hamada M 1995 Statistical tools for the rapid development and evaluation of high-reliability products *IEEE Trans. Reliab.* **44** 187–98
- [49] Feinberg A A and Widom A 1996 Connecting parametric aging to catastrophic failure through thermodynamics *IEEE Trans. Reliab.* **45** 28–33
- [50] Chuang S-L, Ishibashi A, Kijima S, Nakayama N, Ukita M and Taniguchi S 1997 Kinetic model for degradation of light-emitting diodes *IEEE J. Quantum Electron.* **33** 970–9
- [51] Meeker W Q, Escobar L A and Pascual F G 2022 *Statistical Methods for Reliability Data* (Wiley Series in Probability and Statistics) 2nd edn (Wiley) p 659
- [52] Lu C J and Meeker W O 1993 Using degradation measures to estimate a time-to-failure distribution *Technometrics* **35** 161–74
- [53] Yang G and Yang K 2002 Accelerated degradation-tests with tightened critical values *IEEE Trans. Reliab.* **51** 463–8
- [54] Tang L C and Chang D S 1995 Reliability prediction using nondestructive accelerated-degradation data: case study on power supplies *IEEE Trans. Reliab.* **44** 562–6
- [55] Tseng S-T, Hamada M and Chiao C-H 1995 Using degradation data to improve fluorescent lamp reliability *J. Qual. Technol.* **27** 363–9
- [56] Nachlas J A 1986 A general model for age acceleration during thermal cycling *Qual. Reliab. Eng. Int.* **2** 3–6
- [57] Shiau J-J and Lin H-H 1999 Analyzing accelerated degradation data by nonparametric regression *IEEE Trans. Reliab.* **48** 149–58
- [58] Deng A 2006 Research on reliability technology of high-reliability and long-lifetime products *Degree of Philosophy Doctor in Engineering* National University of Defense Technology, Changsha, China
- [59] Deng A M, Chen X, Zhang C H and Wang Y S 2007 A comprehensive review of accelerated degradation testing *Bingong Xuebao/Acta Armamentarii* **28** 1002–7
- [60] Deng A M, Chen X, Zhang C H and Wang Y S 2006 Reliability assessment based on performance degradation data *Yuhang Xuebao/J. Astronaut.* **27** 546–52
- [61] Kececioglu D 2002 *Reliability Engineering Handbook* vol 2 (DEStech Publ) pp 550
- [62] Misra K B 1992 *Reliability Analysis and Prediction: A Methodology Oriented Treatment* (Fundamental Studies in Engineering) (Elsevier) p 889
- [63] Klein J P and Basu A P 1981 Weibull accelerated life tests when there are competing causes of failure *Commun. Statist. Theory Methods* **10** 2073–100
- [64] Kim C M and Bai D S 2002 Analysis of accelerated life test data under two failure modes *Int. J. Reliab. Qual. Saf. Eng.* **09** 111–25
- [65] Pascual F 2008 Accelerated life test planning with independent Weibull competing risks *IEEE Trans. Reliab.* **57** 435–44
- [66] Bunea C and Mazzuchi T A 2005 Bayesian accelerated life testing under competing failure modes *Annual Reliability and Maintainability Symp., 2005 Proc. (30 September 2024)* (IEEE) pp 152–7 (available at: <http://ieeexplore.ieee.org/document/1408354/>)
- [67] Bunea C and Mazzuchi T A 2006 Competing failure modes in accelerated life testing *J. Stat. Plan. Inference* **136** 1608–20
- [68] Zhang Z and Mao S 1994 Statistical analysis of simple step-stress accelerated life-tests with competing causes of failure *Appl. Math. J. Chinese Univ.* **9** 421–8
- [69] Wang B and Wang L 1999 The parameter estimation for progressive stress accelerated life testing of products with competing failure mechanisms *J. East China Norm. Univ. Nat. Sci.* **1** 16–21
- [70] Zhang Z 2002 *Accelerated Life Tests and Their Statistical Analysis* (Beijing University of Technology Press)
- [71] Wang B 2002 Statistical analysis for accelerated life tests with competing causes of failure *Acta Math. Appl. Sin.* **25** 254–62
- [72] Gao W and Zhao X 2006 Statistical analysis of constant stress accelerated life testing of mixture exponential distribution *Mech. Sci. Technol.* **25** 913–6

- [73] Nelsen R B 2006 *An Introduction to Copulas (Springer Series in Statistics)* 2nd edn (Springer) p 269
- [74] Patton A J 2006 Modelling asymmetric exchange rate dependence\* *Int. Econ. Rev.* **47** 527–56
- [75] Breymann W, Dias A and Embrechts P 2003 Dependence structures for multivariate high-frequency data in finance *Quant. Finance* **3** 1–14
- [76] Embrechts P, Höing A and Juri A 2003 Using copulae to bound the value-at-risk for functions of dependent risks *Finance Stoch.* **7** 145–67
- [77] Embrechts P, McNeil A J and Straumann D 2002 Correlation and dependence in risk management: properties and pitfalls *Risk Management* 1st edn, ed M A H Dempster (Cambridge University Press) pp 176–223 (available at: [www.cambridge.org/core/product/identifier/CBO9780511615337A013/type/book\\_part](http://www.cambridge.org/core/product/identifier/CBO9780511615337A013/type/book_part))
- [78] Smith M D 2003 Modelling sample selection using Archimedean copulas *Econom. J.* **6** 99–123
- [79] Patton A J 2012 A review of copula models for economic time series *J. Multivar. Anal.* **110** 4–18
- [80] Li D, Tang X, Phoon K, Chen Y and Zhou C 2013 Bivariate simulation using copula and its application to probabilistic pile settlement analysis *Int. J. Numer. Anal.* **37** 597–617
- [81] Zhang X P, Shang J Z, Chen X, Zhang C H and Wang Y S 2013 Statistical inference of accelerated life test with competing failures based on three-parameter Weibull distribution *Bingong Xuebao/Acta Armamentarii* **34** 1603–10
- [82] Zhang X, Shang J, Chen X and Zhang C 2014 Statistical inference of varying-stress accelerated life test with competing failures based on three-parameter Weibull distribution *J. Mech. Eng.* **50** 42–49
- [83] Zhang X P, Shang J Z, Chen X, Zhang C H and Wang Y S 2014 Statistical inference of accelerated life testing with dependent competing failures based on copula theory *IEEE Trans. Reliab.* **63** 764–80
- [84] Li Y, Pan G, Li Q, Wang C and Hu X 2020 A novel accelerated life-test method under thermal cyclic loadings for electronic devices considering multiple failure mechanisms *Microelectron. Reliab.* **114** 113766
- [85] Pan G, Ding X, Li D, Li Y and Wang Y 2023 A reliability evaluation method of complex electromechanical products based on the multi-stress coupling acceleration model *Eng. Fail. Anal.* **146** 107115
- [86] Zhao W and Elsayed E A 2004 An accelerated life testing model involving performance degradation *Annual Symp. Reliability and Maintainability, 2004—RAMS (30 September 2004)* (IEEE) pp 324–9 (available at: <http://ieeexplore.ieee.org/document/1285469/>)
- [87] Tan Y, Zhang C and Chen X 2011 Analysis of step stress accelerated testing with competing failure modes *Acta Aerodyn. Sin.* **32** 429–37
- [88] Zhang X 2013 Research on theory and methodology of bearing life prediction based on accelerated testing *Degree of Philosophy Doctor in Engineering* National University of Defense Technology, Changsha, China
- [89] Cao Y, Tang J, He P and Cao L 2020 Statistical analysis of constant stress accelerated degradation test in the case of competing failure *J. Hubei Univ. Nat. Sci.* **42** 344–51
- [90] Liu Y, Wang Y, Fan Z, Bai G and Chen X 2021 Reliability modeling and a statistical inference method of accelerated degradation testing with multiple stresses and dependent competing failure processes *Reliab. Eng. Syst. Saf.* **213** 107648
- [91] Chi B, Wang Y, Hu J, Zhang S and Chen X 2022 Reliability assessment for micro inertial measurement unit based on accelerated degradation data and copula theory *Eksploracja i Niezawodnosć—Maint. Reliab.* **24** 554–63
- [92] Yang G-B 1994 Optimum constant-stress accelerated life-test plans *IEEE Trans. Reliab.* **43** 575–81
- [93] Boulanger M and Escobar L A 1994 Experimental design for a class of accelerated degradation tests *Technometrics* **36** 260–72
- [94] Ye Z-S, Chen L-P, Tang L C and Xie M 2014 Accelerated degradation test planning using the inverse Gaussian process *IEEE Trans. Reliab.* **63** 750–63
- [95] Alhadeed A A and Yang S-S 2002 Optimal simple step-stress plan for Khamis-Higgins model *IEEE Trans. Reliab.* **51** 212–5
- [96] Liao C-M and Tseng S-T 2006 Optimal design for step-stress accelerated degradation tests *IEEE Trans. Reliab.* **55** 59–66
- [97] Bai D S, Chun Y R and Cha M S 1997 Time-censored ramp tests with stress bound for Weibull life distribution *IEEE Trans. Reliab.* **46** 99–107
- [98] Tsai T-R, Sung W-Y, Lio Y L, Chang S I and Lu J-C 2016 Optimal two-variable accelerated degradation test plan for gamma degradation processes *IEEE Trans. Reliab.* **65** 459–68
- [99] Wang Y, Zhang C, Zhang S, Chen X and Tan Y 2015 Optimal design of constant stress accelerated degradation test plan with multiple stresses and multiple degradation measures *Proc. Inst. Mech. Eng. O* **229** 83–93
- [100] Wang Y, Chen X and Tan Y 2017 Optimal design of step-stress accelerated degradation test with multiple stresses and multiple degradation measures *Qual. Reliab. Eng. Int.* **33** 1655–68
- [101] Ma Z, Liao H, Ji H, Wang S, Yin F and Nie S 2021 Optimal design of hybrid accelerated test based on the Inverse Gaussian process model *Reliab. Eng. Syst. Saf.* **210** 107509
- [102] Fan T and Wang Y 2021 Comparison of optimal accelerated life tests with competing risks model under exponential distribution *Qual. Reliab. Eng. Int.* **37** 902–19
- [103] Ayasse D and Seo K 2022 Simulation-based search for optimal designs of accelerated life tests through response surface methodology *IJQRM* **39** 137–54
- [104] Kumar J, Kumar K and Ahmad N 2024 Methods of optimal accelerated life test plans: a review *AJTAS* **13** 213–26



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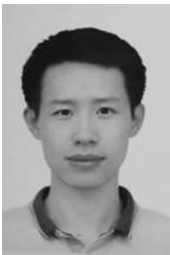


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